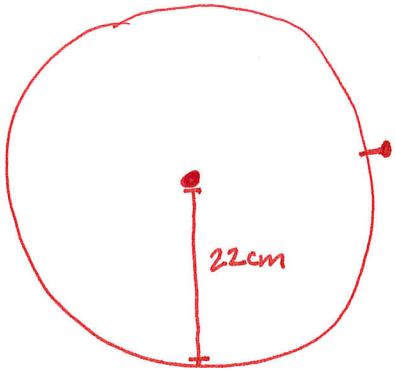
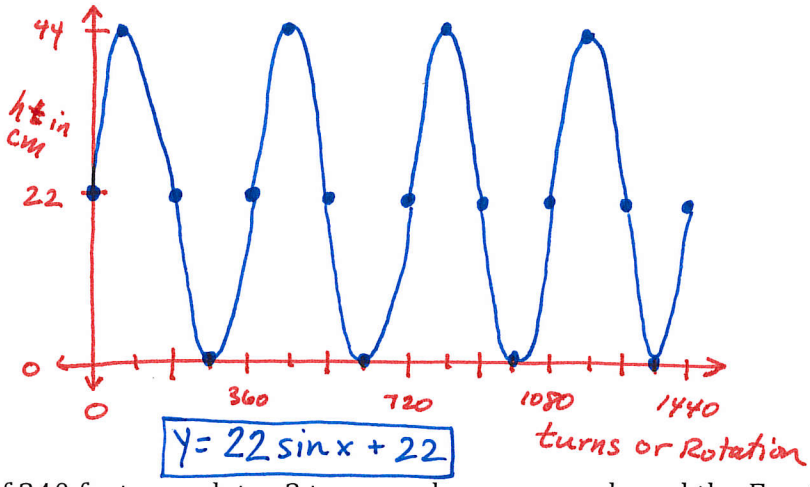
REVIEW OF MODULE 3

EXTENDED RESPONSE

NAME: Key

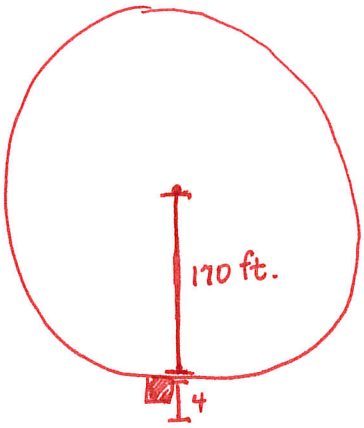
1. A thumbtack is stuck to a bicycle tire. If the tire has a radius of 22 cm, sketch the height of the thumbtack above the ground as the tire rotates counterclockwise through 4 turns. Start your graph when the thumbtack is at the 3 o'clock position. Provide appropriate labels on the graph. Write the equation for the graph.

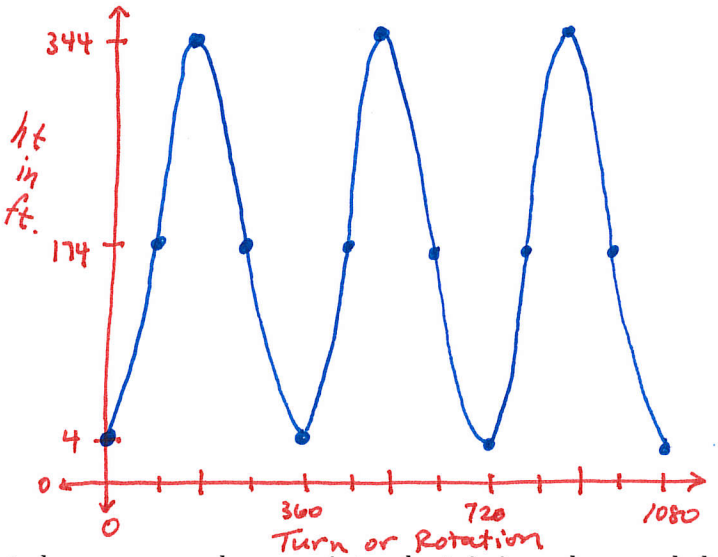




$y = 22\sin x + 22$

2. If a Ferris wheel with a diameter of 340 feet completes 3 turns and passengers board the Ferris wheel at the bottom (which is 4 feet above the ground), create a graph of a function that represents the height above the ground of the passenger car. Provide appropriate labels on the axes.





3. An oscilloscope is a machine that changes sound waves into electric impulses and shows their graph on a monitor. One such graph can be represented by the equation $A(t) = 12\sin(\frac{2\pi}{15}t)$ where t represents time in seconds. What is the period of the function?

$P = \frac{2\pi}{F} = \frac{2\pi}{\frac{2\pi}{15}} = \frac{2\pi \cdot 15}{2\pi} = 15$

4. What is the maximum value of the function: $f(x) = 12 - 3\cos\left(\frac{3\pi}{2}x\right)$?

$f(x) = -3\cos\left(\frac{3\pi}{2}x\right) + 12$

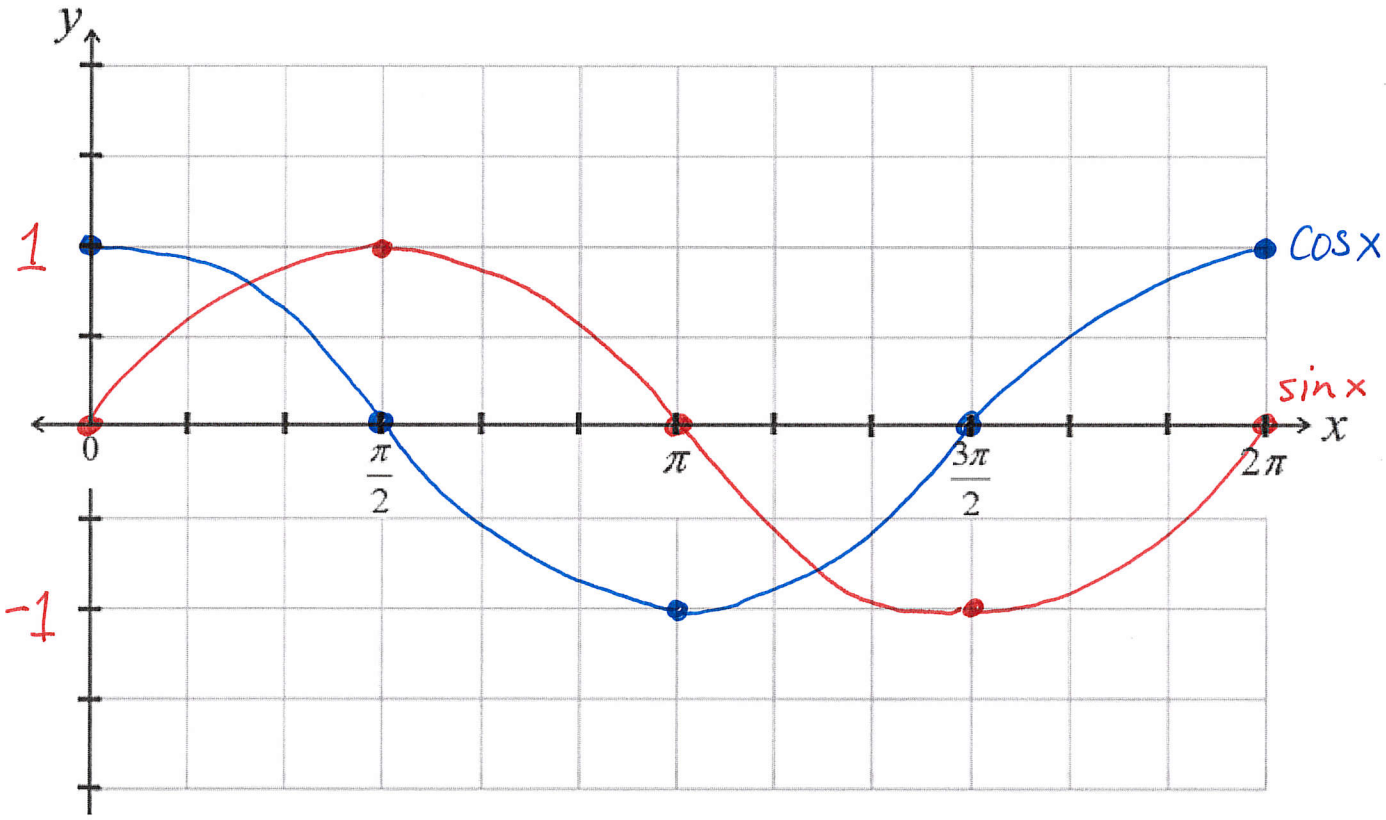
Midline: 12 $\xrightarrow{+3}$ 15 max

$\xrightarrow{-3}$ 9

Amp: 3

Max = 15

5. Using the domain of $0 \leq x \leq 2\pi$, graph the functions $y = \boldsymbol{sinx}$ and $y = \boldsymbol{cosx}$ on the same set of axes below.



a) For what values of x does $sinx = cosx$??

Deg → 45° & 225°

OR

Rad → $\frac{\pi}{4}$ & $\frac{5\pi}{4}$

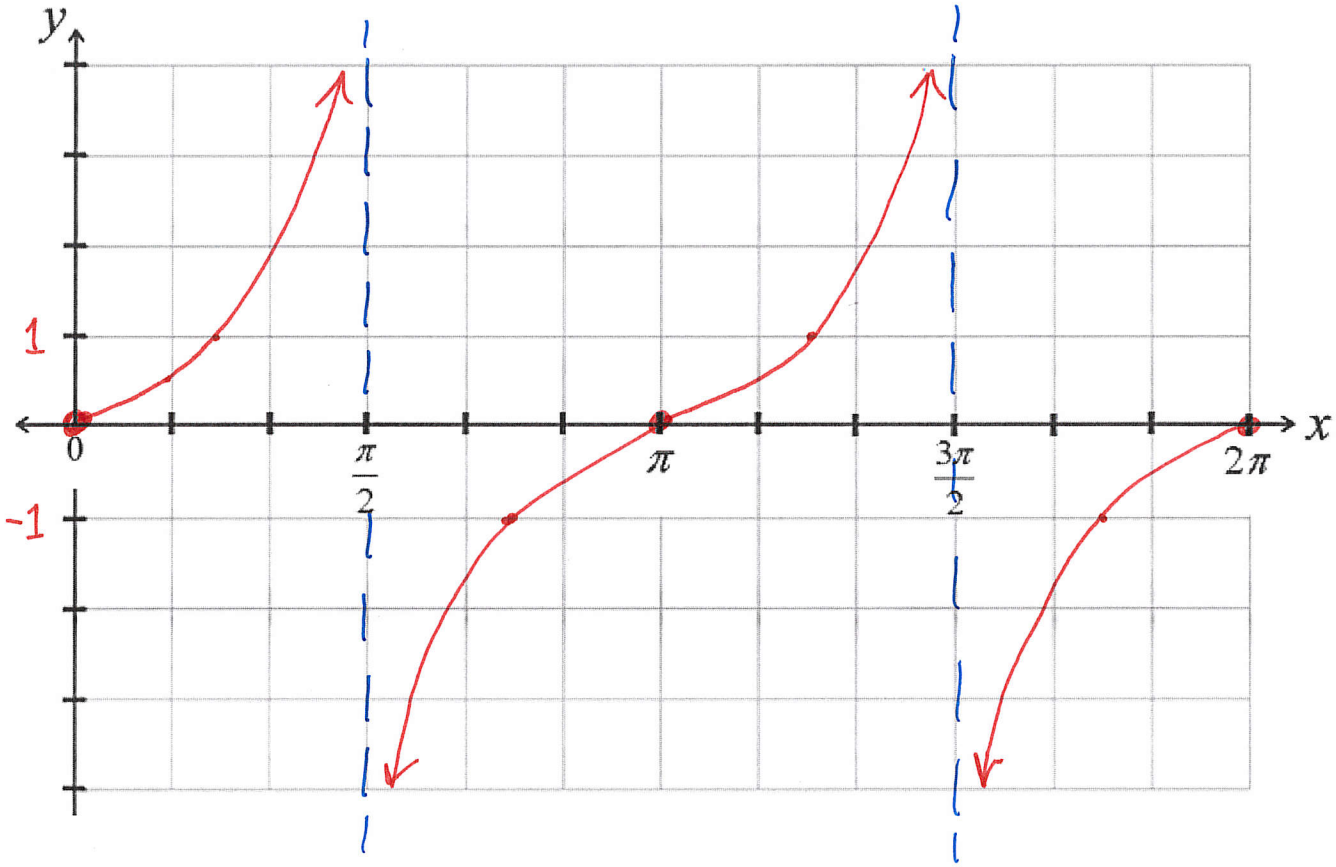
b) In what interval(s) are both $y = sinx$ and $y = cosx$ positive?

Sin → $0 < x < \pi$ Cos → $0 < x < \frac{\pi}{2}$ and $\frac{3\pi}{2} < x < 2\pi$

c) In what interval(s) are both $y = sinx$ and $y = cosx$ negative?

Sin → $\pi < x < 2\pi$ Cos → $\frac{\pi}{2} < x < \frac{3\pi}{2}$

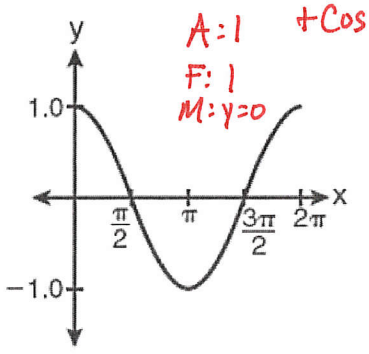
6. Graph $y = tanx$ on the set of axes below:



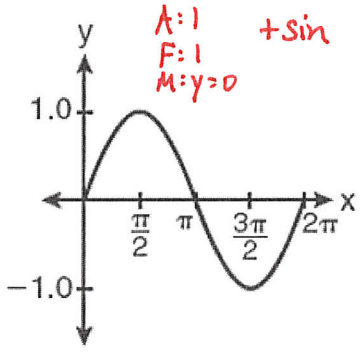
7. Complete the chart below based upon the graphs of $y = tanx$ and $y = sinx$.

Similarities	Differences
Both start, end & have middle points on x-axis	Sin is continuous → tan has asymptotes @ $\frac{\pi}{2}$ & $\frac{3\pi}{2}$ → sin has max @ $\frac{\pi}{2}$ min @ $\frac{2\pi}{2}$

8. Write an equation for each of the functions shown below:

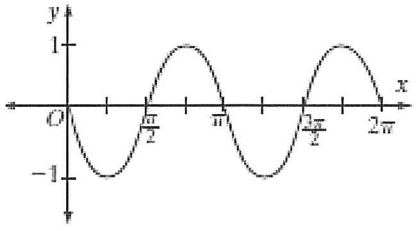


$y = \cos x$



$y = \sin x$

A:1 -sin
F:2
M: y=0



$y = -\sin 2x$

9. Complete the chart below:

θ	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
$sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\varnothing$	0	$\varnothing$	0

10. Convert each of the degree measures below to radian measure.

330^0	$\frac{11\pi}{6}$	270^0	$\frac{3\pi}{2}$
225^0	$\frac{5\pi}{4}$	150^0	$\frac{5\pi}{6}$
90^0	$\frac{\pi}{2}$	315^0	$\frac{7\pi}{4}$

11. Convert each of the radian measures below to degree measure.

$\frac{5\pi}{6}$	150°	$\frac{11\pi}{6}$	330°
$\frac{2\pi}{3}$	120°	$\frac{3\pi}{4}$	135°
$\frac{\pi}{3}$	60°	$\frac{8\pi}{9}$	160°

12. State the **amplitude**, **period**, **horizontal shift**, and **vertical shift** for each of the following:

a) $y = 4\sin(2x + \pi) - 1$

$y = 4\sin\left(2\left(x + \frac{\pi}{2}\right)\right) - 1$

A: 4

$P: \frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4 = P$

HS: $\leftarrow \frac{\pi}{2}$ or 90°

VS: $\downarrow 1$

b) $y = -5\cos(6x - \frac{\pi}{7}) + 2$

$y = -5\cos\left(6\left(x - \frac{\pi}{42}\right)\right) + 2$

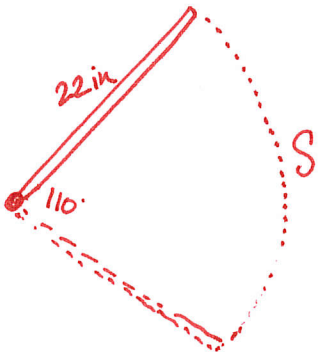
A: 5

$P = \frac{2\pi}{\frac{\pi}{42}} = 2\pi \cdot \frac{42}{\pi} = 84 = P$

H.S.: Right $\frac{\pi}{42}$

V.S: $\uparrow 2$

13. Suppose a windshield wiper arm has a length of 22 inches and rotates through an angle of 110^0. What distance does the tip of the wiper travel, *to the nearest inch*, as it moves from one side of the windshield to the other.



$S = \theta r$

$110^\circ = \frac{11\pi}{18}$ radians

$S = \frac{11\pi}{18}(22)$

$S = 42.23679679\ldots$

$S \approx 42$ inches

14. Verify the Pythagorean identity: $\mathbf{1 + tan^2 x = sec^2 x}$

$$1 + \frac{S^2}{C^2} = \frac{1}{C^2}$$

$$= \frac{C^2}{C^2} + \frac{S^2}{C^2} = \frac{1}{C^2}$$

$$= \frac{S^2 + C^2}{C^2} = \frac{1}{C^2}$$

$$= \frac{1}{C^2} = \frac{1}{C^2} \Rightarrow \sec^2 x = \sec^2 x$$

15. Verify this identity: $\mathbf{sec\theta - sin\theta tan\theta = cos\theta}$

$$\frac{1}{C} - \frac{S}{1} \cdot \frac{S}{C} = C$$

$$\frac{1}{C} - \frac{S^2}{C} = C$$

$$\frac{1 - S^2}{C} = C$$

$$\frac{C^2}{C} = C$$

$$\cos\theta = \cos\theta$$

16. Given that $sin^2x + cos^2x = 1$, and $cosx = \frac{-3}{4}$, find the value of $sinx$

$S^2 + C^2 = 1$

$S^2 + (\frac{-3}{4})^2 = 1$

$S^2 + \frac{9}{16} = 1$

$-\frac{9}{16} \quad -\frac{9}{16}$

$\sqrt{S^2} = \sqrt{\frac{7}{16}}$

$S = \boxed{\pm \frac{\sqrt{7}}{4}}$

17. Write the equation of a graph satisfying all of the following conditions:

- Sine
- Period of 9π → $F = \frac{2\pi}{P} = \frac{2\pi}{9\pi} = \frac{2}{9} = F$
- Amplitude of 7 A
- Shifted 3 down M: $y = -3$

A F M

$y = 7\sin(\frac{2}{9}x) - 3$

18. Graph ONE cycle of: $y = 2\cos(\frac{1}{2}x - \pi)$

$y = \underset{A}{2}\cos(\underset{F}{\frac{1}{2}}(x - 2\pi))$ → 360°

A: 2

F: $\frac{1}{2}$

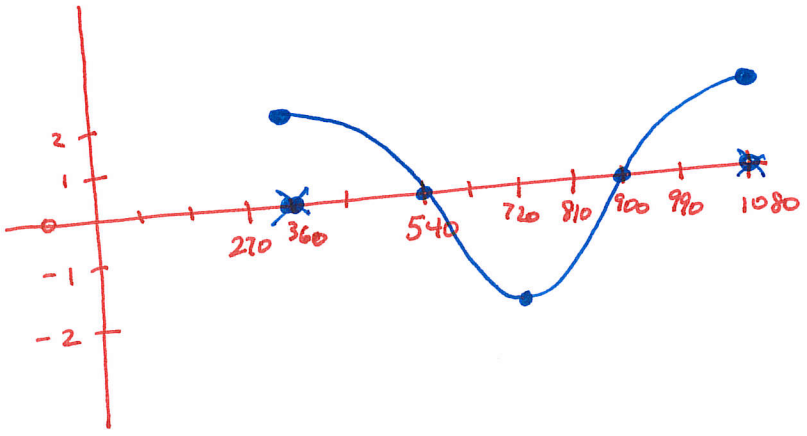
P: $\frac{360}{\frac{1}{2}} = 720$

M: $y = 0$

→ 360

0 ~~180~~ ~~360~~ ~~540~~ ~~720~~

360 540 720 900 1080



19. Graph ONE cycle of: $y = -4\sin(3x + \pi)$ = $-4\sin\left(3\left(x + \frac{\pi}{4}\right)\right)$ ← 45°

V.F. ∫

A: 4

F: 3

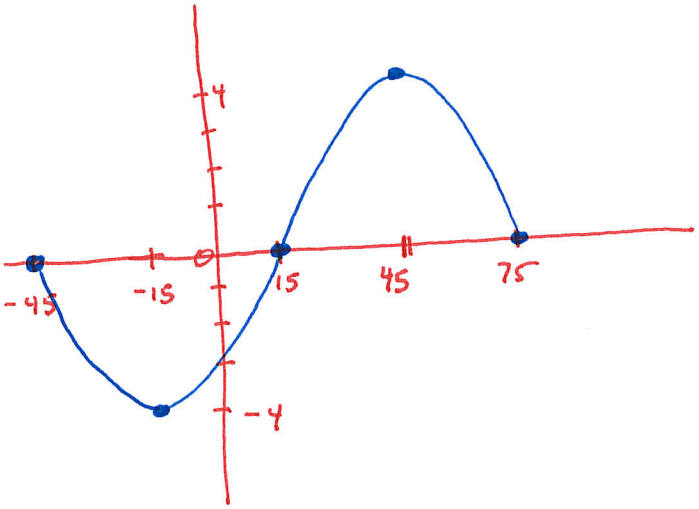
P: $\frac{360}{3} = 120$

M: y = 0

← 45°

~~0~~ ~~30~~ ~~60~~ ~~90~~ ~~120~~

-45 -15 15 45 75



20. Graph ONE cycle of: $y = -3\cos(6x + \pi)$

$y = -3\cos\left(6\left(x + \frac{\pi}{6}\right)\right)$ ← 30

-cos ∩

A: 3

F: 6

P: $\frac{360}{6} = 60$

M: y = 0

← 30

~~0~~ ~~15~~ ~~30~~ ~~45~~ ~~60~~

-30 -15 0 15 30